Measuring the Earth-Moon Distance by Using Geometric Modeling from the Lunar Eclipse

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Abstract

The Earth-Moon distance was measured from the lunar images that were taken during the Total Lunar Eclipse phenomenon on December 10, 2011. With the basic geometric method, the measured value is 373,550 kilometers, difference from the reference value (397,254 km) [1] by around 6%.

Introduction

Nowadays, we know that the best way to measure the Earth-Moon distance is to use laser measurement called Lunar Laser Ranging (LLR) but this method has been used only in recent years after the Apollo 11 mission successfully installed a laser reflector on the Moon in 1969. Nevertheless, Greek astronomers can measure this approximate value since more than 2,000 years ago by carefully observing the lunar eclipse and using only basic geometric method for analysis. So, our study is to follow the ancient method by using data from the total lunar eclipse that occurred on December 10, 2011.

Method

When we plot the position of the Sun, Earth and Moon during total lunar eclipse phenomena, we can get the geometric model as shown in Figure 1.

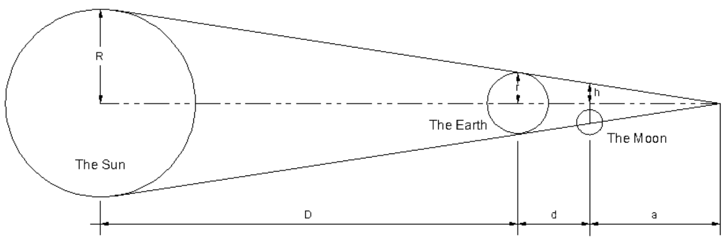
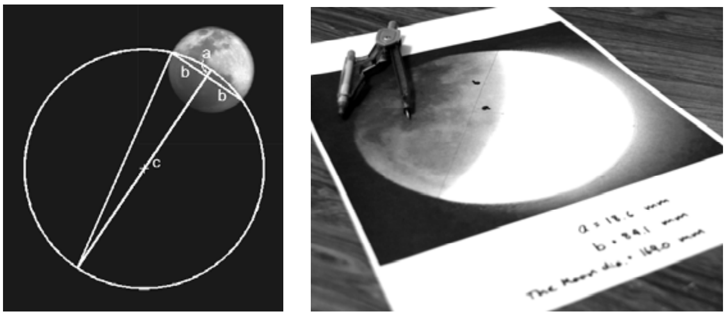


Figure 1 Geometric modeling of the lunar eclipse phenomena.

With a similar triangle's law, we can take a set of equations and reduce to only one equation that relates to the Earth-Moon distance value (**d**) as shown below:

$$d = \frac{D(r-h)}{(R-r)} \qquad \text{-------} \qquad (1)$$

Assume that we already knew the value of the Sun-Earth distance (**D**), the radius of the Sun (**R**) and the radius of the Earth (**r**), while the radius of the earth's shadow with respect to the Moon distance (**h**) as our unknown value. From this, we plan to measure the radius of the earth's shadow by using the lunar images that were taken during the eclipse process. From Figure 2 (a), if we imagine the curve of the Earth's shadow is extended to a complete circle and try to make a "chord" and some "triangles" in the circle of the earth's shadow, we can create the equations that relate to the radius of earth's shadow (**h**) in term of ***a*** and ***b*** values. The result is shown in equation (2). Then, we measured the value of ***a***, ***b*** and lunar diameter ($\mathbf{d_m}$) from the image as shown in Figure 2 (b).



(a) (b)

Figure 2 (a) Diagram of the Earth's shadow circle and a chord for deriving the equation (2) (b) Measure the value of ***a***, ***b*** and lunar diameter ($\mathbf{d_m}$) in millimeters from the images.

$$h = (a + \frac{b^2}{a}) / 2 \qquad \text{-------} \quad (2)$$

The calculated radius of earth's shadow (**h**) in millimeters from equation (2) can be converted to kilometers by using comparison between the measured lunar diameter ($\mathbf{d_m}$) in millimeters to the actual lunar size in kilometers and use this scale for changing the radius of earth's shadow (**h**) to kilometers. Finally, substitute the radius of earth's shadow (**h**) to the equation (1) to obtain the Earth-Moon's distance in kilometers.

Result

After we use this method to determine the Earth-Moon's distance from a set of lunar images, the average value of the Earth-Moon's distance is 373,550 kilometers, difference from the reference value (397,254 km) [1] by around 6%

Conclusion

The obtain value from this method is quite precise. We have only small error around 6%. We think it may cause of the difficulty to identify the edge of the earth's shadow on the lunar images.

Reference

[1] Heavens-Above. **Moon Data**. Available Source: http://www.heavens-above.com/, Jan. 9, 2012.