The Study of Trojan Motion Using N-body Simulation

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Abstract

The purposes of this research are (1) to be able to derive the motion of an object in the solar system, and (2) to be able to simulate the motion of trojan asteroids in some studied cases. After simulating the motion of an object in the solar system, the position data was compared to JPL's Horizon. The simulation produced low error, with a mean error of 0.0333% and a standard deviation of 0.119%. Mercury has the highest error of 0.17400%, while Mars has the lowest error of 0.00108 %. According to experiments, trojan motion is influenced by its velocity and mass. The trojan's orbit around lagrage point, L4, expands as its velocity increases. But as its mass increases, trojan's orbit become more stable.

Introduction

Trojans are celestial bodies that orbit near the L4 or L5 Lagrange point of stability. Newton's law of gravitation states that all objects interact with one another. As a result, each object must withstand multiple forces. One method for determining the trojan orbit is to use N-body simulation. The N-body problem is a complex and difficult problem to solve with an accurate solution. Computer programming has become one of the methods for assisting in the evaluation of the N-body problem. The leapfrog method, also known as the "kick-drift-kick" method, is one of the efficient numerical methods for evaluating the N-body problem. The simulation of trojan motion in Python 3 is developed in this work using the Leapfrog method as well as the N-body problem.

Summary

PART 1: The solar system simulation

An N-body simulation is created in Visual Studio Code using Python 3 and the NumPy and Matplotlib libraries. The simulation employs Newton's Law of Gravitation and is calculated using the Leapfrog method, which is stable for oscillatory motion, in this case planetary motion.

$$\vec{F} = \frac{Gm_1m_2}{r^3}\vec{r}$$

$$\vec{v}\left(t+\frac{\Delta t}{2}\right) = \vec{v}(t) + \vec{a}(t)\frac{\Delta t}{2}$$

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}\left(t+\frac{\Delta t}{2}\right)\Delta t$$

$$\vec{v}(t+\Delta t) = \vec{v}\left(t+\frac{\Delta t}{2}\right) + \vec{a}(t+\Delta t)\frac{\Delta t}{2}$$

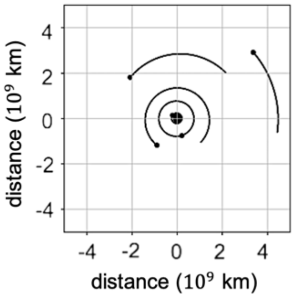

Figure 1: The created solar system simulation

Using positions and velocities of the objects in the solar system on January 1, 2021, 0.00 UT from JPL's HORIZON as the initial conditions, then run the simulation as shown in figure 1. After that comparing the simulated positions to database.

Table 1 The comparisons of mean percentage error all over 149 days in simulation.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Error (%)	0.17400	0.03827	0.04550	0.00108	0.00318	0.02660	0.00147	0.00927

As a result of the comparison, the mean error is 0.0333%, and the standard deviation is 0.119%. Mercury has the highest error of 0.17400%, while Mars has the lowest error of 0.00108%. As shown in table 1.

The fact that Mercury has the highest error is likely due to the simulation using classical mechanics, but Mercury orbits in a strong gravitational field, so there must be a relativistic effect causing more error. This means that the simulation works well in solving the N-body problem if none of the objects is too close to each other.

PART 2: Derive the trojan motion

The simulation from PART 1 is converted into a three-body problem model. Then apply the positions of all the object included trojan. Simulate and collect the positions in all timesteps for further analysis.

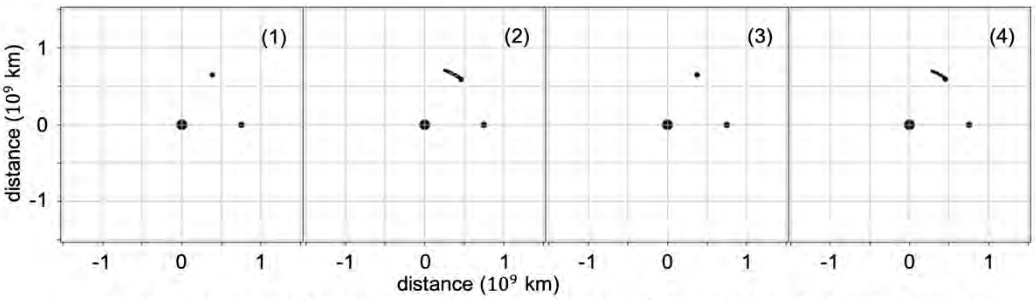

Figure 2: Diagrams of the trojan's path with (1) massless (2) massless with 0.5% higher velocity (3) 80% mass of Jupiter (4) 80% mass of Jupiter with 0.5% higher velocity

The results of this simulation show that increasing trojan's velocity causes trojan's orbit to grow larger, but increasing trojan's mass may affect Jupiter's motion, making trojan's orbit more stable. As shown in the figure 2 above.

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