W14b Higher Order Methods for Numerical Simulations Including Self-Gravity

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Modern numerical simulations often employ higher order methods to achieve higher spatial and temporal accuracy and track wave propagation. Since self-gravity is important in many astronomical environments, it should also be evaluated accurately. However, to our knowledge, numerical algorithms for self-gravitating hydrodynamics are limited to second-order accuracy in space. In this poster we introduce a method to derive the gravitational acceleration per unit volume ($\rho\mathbf{g}$) and gravitational energy release per unit volume ($\rho\mathbf{v}\cdot\mathbf{g}$) at fourth-order accuracy in space for a given cell average density (ρ) on a uniform Cartesian grid and numerical mass flux ($\rho\mathbf{v}$) across a cell surface.

Our scheme has the following features: (1) the truncation error is proportional to h^4 for both $\rho\mathbf{v}$ and $\rho\mathbf{v}\cdot\mathbf{g}$, where h denotes the unit cell width; (2) the total linear momentum is conserved exactly (to round-off error); (3) the total energy (including gravitational potential energy) is conserved without truncation error; and (4) all terms are given explicitly and do not require iteration (except for solving the Poisson equation).

We also present several numerical examples including (1) an error analysis for a self-gravitating spherical body with an overlying, low mass, power law envelope, (2) the advection of a self-gravitating sheet, (3) the propagation of a gravito-acoustic wave, and (4) the hydrostatic equilibrium of an $n = 5$ polytrope. These tests illustrate that our scheme is free of artificial gravitational heating and cooling. This work has been assigned a document release number LA-UR-23-26228.